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Fatigue curves without testing

When you don't have the time to wait out a million-cycle fatigue test, you can approximate the results in a jiffy with handbook data.

By Peter R. Weihsmann, Senior Engineer
Reliance Electric Research Center, Cleveland, Ohio

Fatigue curves are costly and time-consuming to generate by actual testing. Fortunately, there are theoretical techniques available for approximating the shape of a fatigue curve. Two such short cuts, the "method of universal slopes" and the "four-point correlation" method, have been known for years. Each, however, needs lengthy calculations, from data that are not always readily available.

An alternative method, based on "four-point correlation" theory, allows fatigue curves to be constructed using only handbook data. This technique correlates reasonably well with real fatigue data for most steels, magnesium, aluminum, and copper. It may work for other metals, too.

Four points, two curves

The theory is based on the premise that total strain in a fatigued specimen is the sum of the elastic and plastic strains. Both the elastic and plastic strain-vs-life relationships are essentially linear on log-log graphs. From the arithmetic summation of these two curves, a fatigue life curve for total strain is constructed.

A step-by-step procedure for locating the end-points of both the plastic and elastic curves from handbook data, and then approximating the fatigue curve, is presented here.

Four points are needed to construct the two curves (See graph 1). Points A and B are used to locate the elastic strain curve. The approximated fatigue curve drawn from these curves is shown.

Although this technique was developed in terms of strain, most fatigue and tensile data are expressed as a function of stress — at least, below the yield strength. Thus, instructions for constructing the fatigue curve are presented here in terms of strain as well as equivalent elastic stress.

Picking Points A through D

A portion of the theory behind this technique assumes that the simple tensile test represents one-fourth of a single, completely reversed fatigue cycle — the peak positive value of the applied stress. Ultimate tensile strength, for example, becomes the stress to cause failure during the first fatigue cycle.

Two points, A and B, used in the construction of the fatigue curve are taken from tensile-test data. Thus, these points are plotted on a vertical axis corresponding to the peak loading during the first cycle. This point is at $N = 1/4$, where N is the number of complete fatigue cycles.

The four points used to construct the elastic and plastic strain curves are found as follows:

- *Point A* (in terms of equivalent stress) is simply the ultimate tensile strength of the metal, plotted on the vertical axis of the graph at $N = 1/4$. Since ultimate tensile strength is in units of stress, this value would be divided by Young's modulus for the metal if the curve is being constructed in terms of strain. Point A is the left-hand locator for the elastic strain curve.
- *Point B,* the right-hand locator of the elastic curve, is defined as the fatigue-endurance limit, if the metal has one; otherwise, Point B is the endurance strength. Some ferrous alloys have an endurance limit, a stress level below which fatigue failure will never occur, regardless of number of cycles. This is generally around 10^7 or 10^6 cycles, at which point the fatigue curve approaches zero-slope, or a horizontal line.

Many metals, particularly those that don't work harden, have no detectable endurance limit. Their long-life fatigue curves never become truly horizontal. For these metals, a pseudo-endurance limit, called endurance strength, is reported. Usually, this value is defined as the failure stress at some large number of cycles — 10^7 to 10^{10}.

Point B can also be obtained from tensile-test data — a virtue of this technique, since handbook values for fatigue-endurance strengths or limits are often not available. Use graph 2 to find the fatigue-endurance value from

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The fabricated fatigue curve

To draw the graph in terms of stress:

Point A = ultimate tensile strength

Point B = endurance strength or endurance limit

Point C = Log strain at tensile fracture × Young's modulus — see text

Point D = 10^4 cycles

To draw the graph in terms of strain:

Point A = ultimate tensile strength ÷ Young's modulus

Point B = endurance strength or limit ÷ Young's modulus

Point C = Log strain at tensile fracture — see text

Point D = 10^4 cycles

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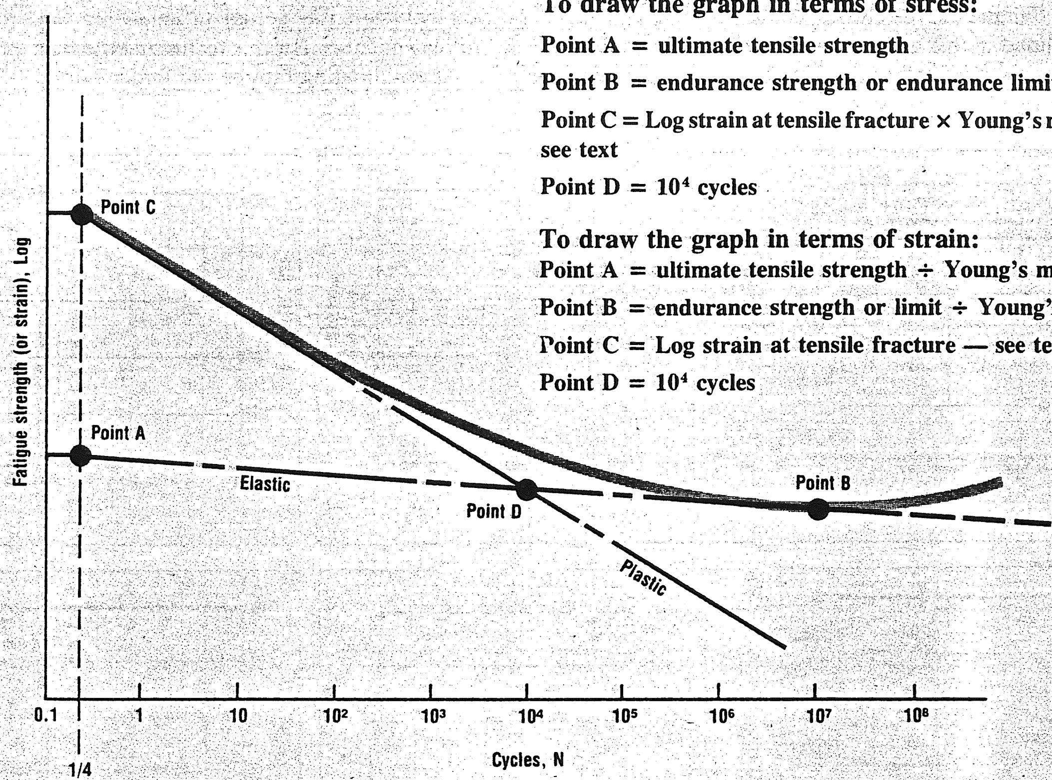

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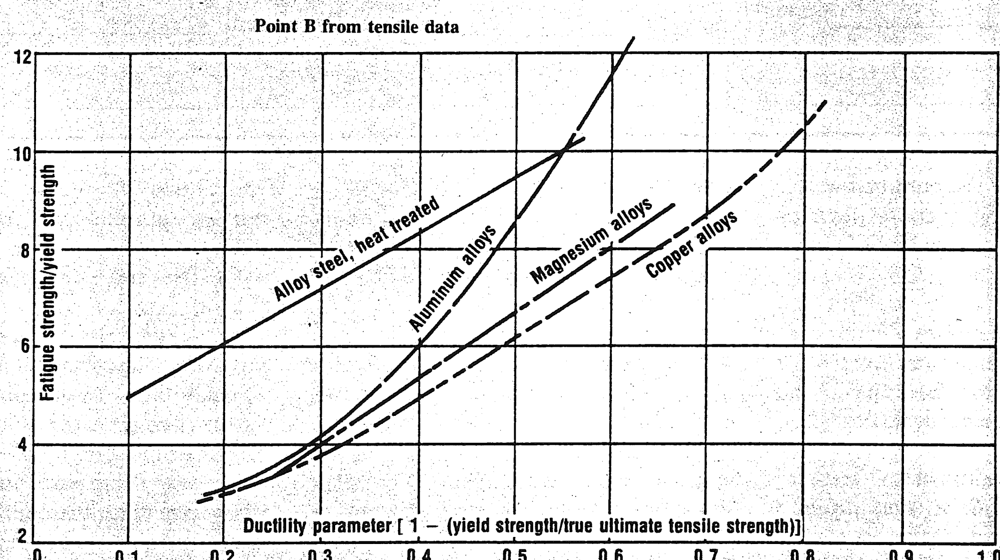
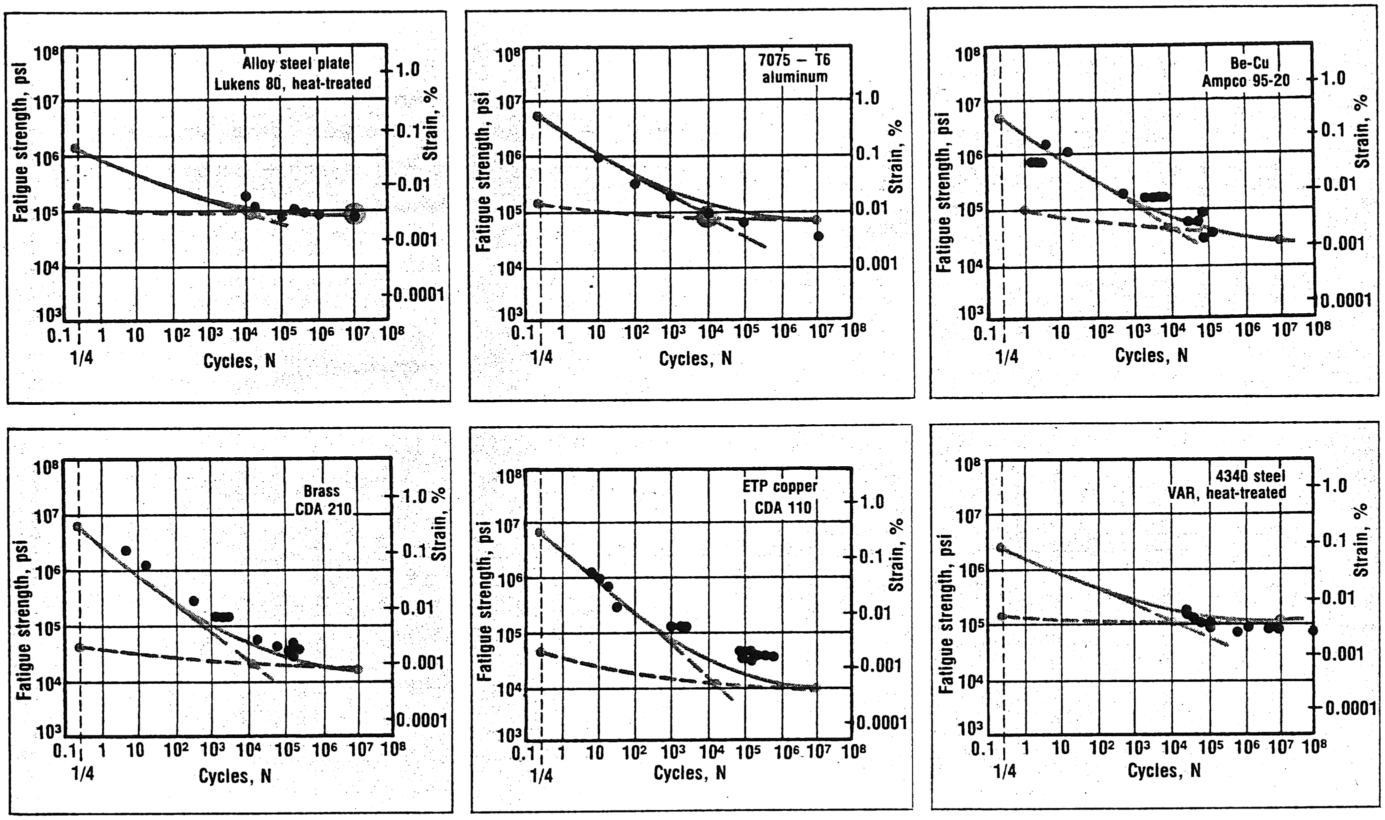


yield strength and true ultimate tensile strength for the material. Simply calculate the "ductility parameter" from handbook tensile data using the equation on the horizontal axis of the graph. Then find the "endurance-to-yield" strength ratio for the appropriate material. Multiply this ratio by "yield strength" to find the endurance value, which is Point B.

Beyond Point B, the ratio of ultimate tensile strength to yield strength can be used to approximate the slopes of the long-life portion of the fatigue curve. According to many researchers, a ratio greater than 1.2 suggests that the material strain hardens sufficiently to produce a pronounced endurance limit value, and the curve assumes a zero slope. For ratios less than 1.2, however, the curve will continue to drop beyond

How accurate?

These six graphs were prepared by the author from actual fatigue-test data (black dots) and from handbook tensile data (colored dots). Fatigue curves constructed according to the techniques outlined in this article are shown in solid color. Elastic and plastic strain curves used in the construction of the fatigue curves are dashed. While this is no substitute for thorough, conventional fatigue testing, reasonable correlation between the actual fatigue data and the simulated curves indicates that this technique can be a quick short-cut for approximating fatigue-life information.



Point B. The lower the ratio below 1.2, the further the fatigue curve deviates from a horizontal, zero-slope line beyond Point B.

Since both endurance strength and endurance limit are reported in terms of stress, this value must be divided by Young's modulus for the metal if the fatigue curve is being constructed in terms of strain.

• *Point C* is a value known as "fracture ductility." If natural, or true, strain at fracture for a simple tension test is known (which would be the distance between gage points at fracture divided by initial gage length), fracture ductility is the natural log of this value.

In most cases, however, reduction of area for a simple tensile test is given in handbooks. In such cases, fracture ductility, e_f is estimated,

$$e_f = ln\frac{100}{100 - R_a}$$

where R_a is reduction of area in %.

Since fracture ductility is in units of strain, this value must be multiplied by Young's modulus to obtain Point C in terms of stress. In all cases, Point C is also plotted at $N = 1/4$.

• *Point D* is defined as the intersection of the plastic and elastic curves at 10^4 cycles. (According to the theory of "universal slopes," elastic and plastic strain curves intersect at $N = 10^4$). Thus, locate Point D on the elastic curve and draw the plastic curve between Points C and D.

Now the fatigue curve can be drawn as the arithmetic summation of the elastic and plastic lines.

How accurate?

To demonstrate the validity of the method described here, actual fatigue test results for various steel, aluminum, and copper alloys were compared with curves approximated from handbook data.

Of the six comparisons shown, fatigue data for steels and aluminum were taken from published sources. The measurements for copper fatigue are original, taken from tests on simulated squirrel-cage rotor, bar-to-end ring joints for induction motors.

Because these parts had been brazed prior to testing, the copper fatigue test data were assumed to represent essentially annealed material. Traceability of the data is not "ideal" in these cases, since handbook tensile data for the approximated curves were selected for truly annealed materials. Nevertheless, correlation between fatigue test data and the curves drawn from annealed tensile data is quite good, indicating that this technique appears to be perfectly acceptable for copper alloys as well. ■